The Cosmological Constant Problem as Transitioning Multi-Space

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Abstract

Resolution of the cosmological constant must maintain the mathematics of wave function, as well as Einstein's field equations. However, a fundamental break from convention is justified, in order to cancel the mathematical discrepancy that exists between the vacuum-energy density in general relativity (GR) and the much greater zeropoint-energy value, as calculated in quantum field theory (QFT).¹ This article does maintain conventional models. The paradigm shift is from the conventional idea of a constant \mathbb{R}^4 spacetime at every scale (from microspace to macrospace), and instead proposes that dimensions approaching quantum scales increase to a higherdimensional \mathbb{R}^5 space and that dimensions approaching the cosmic event horizon decrease to a lower-dimensional \mathbb{R}^3 space (\mathbb{R}^4 spacetime is just one phase of this transitioning multispace). This is only hypothetical, yet it explains so many mysteries of cosmology in the simplest and most parsimonious format (in fact, in one single sentence). In wave function, superposition can be alternatively interpreted as a transition (or divergence) of \mathbb{R}^n , as observed from an \mathbb{R}^4 spacetime onto a higher-dimensional \mathbb{R}^5 space at scales approaching QFT. Time, position, energy and momentum become plural, and a range of \mathbb{R}^5 space is viewed as a single instant until "collapsed" (mapped) onto our familiar classic \mathbb{R}^4 spacetime. In GR, \mathbb{R}^4 spacetime transitions (or converges) to a point at the cosmic event horizon, to a diminished \mathbb{R}^3 space. This converging affect appears equivalent to acceleration, as units of time in velocity are decreasing with larger scales. Thus, Λ in GR is only illusional and therefore cancelled.

1. INTRODUCTION

A transitioning multi-space, from quantum scale to the cosmic event horizon, is suppositional. However, as diminishing degrees of freedom (d.o.f.) and increasing entropy become propagated within the Higgs field, it seems reasonable to consider that parameters of information are correspondingly diminished within transitioning \mathbb{R}^n vector fields. These effects can be modeled as partial mapping with consideration to reduced parameters in the image range, as compared with the codomain.

$F: \mathbb{R}^n \to /\mathbb{R}^{n-1}$

In section 5 is described the mechanism which delineates between each phase of \mathbb{R}^n space as "Parametric Validation Reinforcement Loops" (PVRL), an iterated process of conscious binary gnomonic mapping of higher-dimensional topology onto biased eigenstates (and subsequent propagation within the Higgs field). At each iteration, symmetry becomes more broken, and geometric parameters become more constrained.

2. TRANSITIONING MULTI-SPACE, FROM QFT TO GR

This theory proposes that scales approaching QFT increase to a higher-dimensional \mathbb{R}^5 space, and that scales approaching the cosmic event horizon decrease to a lower-dimensional \mathbb{R}^3 space (\mathbb{R}^4 spacetime is just one phase of this transitioning multispace).² In the wave function, superposition can be alternatively interpreted as a dimensional divergence of \mathbb{R}^n , as viewed from an \mathbb{R}^4 space

to an \mathbb{R}^5 space. In GR, \mathbb{R}^4 spacetime is decreasing (or converging) to a point at the cosmic event horizon (at a diminished \mathbb{R}^3 space). (See figure 1.) Thus, scale magnitude is inversely proportional to dimensionality:

<u>Eq. 1</u>: $||\mathbf{x}||_2 \propto \mathbb{R}^{-n}$

3. WAVE FUNCTION IN HIGHER DIMENSIONS

A wave function in higher dimensions is proposed as an alternative to superposition.³ Observations from \mathbb{R}^4 spacetime to quantum scales in \mathbb{R}^5 hyperspace would appear as a range of energy, momentum, position, and time. A divergence in the time dimension (*div t*) can be expressed as:

Eq. 2:
$$div t = \frac{\partial t_1}{\partial x_1} + \frac{\partial t_2}{\partial x_2} + \cdots + \frac{\partial t_n}{\partial x_n}$$

Particles in multiple time and positions would be observed simultaneously, such that a range of position and time would appear as a dense mass in a single moment (like an orbital), similar to observing a time-lapsed photographed image. Imagine an \mathbb{R}^2 disk observing an \mathbb{R}^3 sphere: An \mathbb{R}^3 curve is of *x*, *y*, *z* coordinates. However, when this \mathbb{R}^3 geometry becomes gnomically projected/mapped onto the disk, it collapses to an \mathbb{R}^2 parametric reduced curve of *x*, *y* coordinates. To view \mathbb{R}^n divergence in the time dimension is to view the past, present, and future in a single instant (along with its associated positions, energies, and plural momentum) appearing as a semi-dense **solid.** So, a projectile will be viewed as an arc. By extension, an orbit will appear as a circle. Note that this theory correctly predicts orbital scattering to be more dense toward the center, as the orbit paths are more frequent, which increases the probability density function (PDF). By further extension, consider observations from \mathbb{R}^5 geometry of x_1 , x_2 , x_3 , x_4 , x_5 collapsed (gnomically mapped) onto \mathbb{R}^4 geometry of x_1 , x_2 , x_3 , x_4 : An \mathbb{R}^5 projectile might appear as a partial torus, and so $\mathbb{R}^5 \Psi 0$ might appear as an s-orbit (with density increasing toward the center). (See figure 2.)



FIG. 1: Gnomonic mapping, in phases, from QFT (at \mathbb{R}^5 space) onto GR (at \mathbb{R}^3 space), with classic \mathbb{R}^4 spacetime intermediately.





 \mathbb{R}^n time divergence in QFT challenges the particle mass in de Broglie's λ by substituting a unit of length, such as r_e (electron radius) or r_o (atomic radius) instead of mass. Hydrogen atom wave function in summation form:

$$\underline{\operatorname{Eq. 3:}} \Psi_{\vec{k}(\vec{r})} = e^{i\vec{k}\cdot\vec{r}}$$

Using $p = \hbar k$ for momentum, the dominant wave function $Psi_{\vec{k_0}}$ includes wave vector (Eq. 4)

$$\vec{k_0}: k_0 = \frac{2\pi}{\lambda_0}$$

This use of scale, instead of mass, for λ resolves the following with greater parsimony:

- The use of scale, in massless photons.
- Electron decay, by understanding electron shells as \mathbb{R}^5 topology mapped onto \mathbb{R}^4 spacetime.

- Quantum "time travel" interaction with past and future, as divergence in the time dimension (viewed from R⁴ to R⁵).
- Backward causation.
- Gaps between orbitals.

(See figure 3.)



FIG. 3: Atomic diameter viewed in time divergence.

Resolution to the Cosmological Constant Problem: Λ in QFT

In equation 1, scale (from QFT to GR) is shown with a corresponding inverse dimensionality:

$||\mathbf{x}||_2 \propto \mathbb{R}^{-n}$.

Similarly, in equation 4, atomic or particle radius is used in λ instead of mass. Thus, λ is inversely proportionate to scale: $\lambda \propto r^{-e}$.

Thus, Λ is also inversely proportionate to scale, measuring with increasing values (range), as volume decreases: $\Lambda \propto r^{-e}$

To reiterate, a transition from \mathbb{R}^4 to \mathbb{R}^5 space, at quantum scales is proposed. Thus, a range of time, energy, position, and momentum are viewed in a single instant (alternative to superposition). If this supposition (that quantum scales exist in higher dimensions than the observer) is correct, then a range of energy, time, momentum, and position are observed and measured (at \mathbb{R}^5), as a single event from the observer's perspective (\mathbb{R}^4). Thus, the total energy in a volume of empty space is significantly less than the sum of its measured units of energy! This is due to overlapping of units (range) represented in each measurement. (Each measurement is a sum of multiple times and positions, viewed in a single flash, in higher-dimensional space.) (See figure 4.)

Since the measurements of energy e in each unit represent a sum of n units, the total energy E_0 is equal to the sum of individual volume measurements divided by n:

$$\underline{\operatorname{Eq.5}}: E_0 = \frac{1}{n} \sum_{i=1}^n e_i$$

A test to demonstrate this " E_0 range theory" would be to construct multiple points of virtual photons, using two disks with spiral grooves, aligned in opposite amplitudes, (space at min nanometers) to function as Casimir plates. Validation would be the result of energy e_n that is greater than the mean of

its local neighbors within *x* range on nanospace:

$$\underline{\operatorname{Eq. 6}}: S' = \{[e_{n-x}, \dots e_{n+x}]\},$$

<u>Eq. 7</u>: x = range of local vacuum states (undetermined)

Eq. 8:
$$e_n > \overline{S'}$$

(See figure 4.)

4. RESOLUTION TO THE COSMOLOGICAL CONSTANT PROBLEM Λ IN GR

The Flattening Effect of PVRL in GR (2)

If this supposition of transitioning \mathbb{R}^n multispace is correct, then observations (from \mathbb{R}^4 spacetime) of galaxies (at \mathbb{R}^3 space) should appear in a dimensional (\mathbb{R}^n) convergence (converging to a point at the cosmic event horizon). This convergence can be graphically represented in a 2D Euclidean coordinate system with converging number lines. (See figure 5). Although they appear similar to log transformations, they represent an actual \mathbb{R}^n



FIG. 4: Overlapping ranges of measured vacuum energy.

dimensional convergence. The rate of convergence corresponds to the cosmological constant Λ , the 2nd derivative of the Hubble constant, $\frac{\ddot{a}}{a} = H_{\Lambda}^2$, in respect to universal expansion. In rotational curves, Λ is applied to r in local systems.

The Galaxy Rotational Curve, Resolved as Transitioning from \mathbb{R}^4 Spacetime to \mathbb{R}^3 Space

(Refer to figure 6.) A theoretical galaxy rotation curve, as predicted by Kepler's law, is shown next to a typical observed galaxy rotation curve. The apparent defiance of the inverse-square-law can be explained, alternatively to "Dark Matter," as a conventional orbital system viewed in converging dimension of time: As r increases, intervals of Δ v/d converge within the local system.







FIG. 6: Converging dimensional space in GR.

The Hubble Constant, Viewed in Converging Dimensions

(Refer to center of figure 6.) The original regression line of the "Hubble Constant," from 0 mpc to 45 mpc, is graphed (left) within the proposed converging dimension of time. A linear increase of velocity (acceleration) is apparent. The same graph is shown (right) with a coordinate transfer to uniform time intervals, demonstrating a decreasing rate of acceleration approaching a constant.

Constant Universal Expansion, Viewed in Converging Dimensions, Appears as Acceleration

(Refer to bottom of figure 6.) An equivalency is established between acceleration, as viewed in uniform dimension units, and velocity, viewed in converging dimension units. The graph on the right appears to be acceleration. However, a coordinate transfer would show a constant velocity. The resolution of Λ is the recognition that accelerated expansion is an illusion of converging time dimension x_0 , thus cancelling Λ . We are left with the original form of:

$$\underline{\text{Eq. 9:}} R_{\mu\nu} - \frac{1}{2}Rg_{u\nu} = \frac{8 \times \pi G}{c^4}T_{\mu\nu}$$

This principle is analogous to viewing railroad tracks in 2D converging-point perspective: Imagine the observer with a reference clock, measuring some motion with velocity (v) across (x). (See figure 7.) In time-dimensional divergence, the clock will measure each successive (d/v) with apparent decreasing time intervals, according to the equation:

$$\underline{Eq. 10:} t_p = \frac{t \bot}{1 + (d^{\frac{1}{2}} * K)}$$



FIG. 7: Converging dimension of time.

 $t \perp = d/v_1$ at the observer's clock

d = distance from observer to event d/v_2

 t_{ρ} = the resulting converged time interval of event d/v₂

5. PARTIAL MAPPING MODEL OF TRANSITIONING MULTI-SPACE

Let open set p_1 be a partial nonconformal gnomic map of sphere S onto a tangent plane.

Eq. 11:
$$S \in \mathbb{R}^{3}_{def} = (x, y, z) \in \mathbb{R}^{2} : ||x^{2} + y^{2} + z^{2} = r||$$

<u>Eq. 12</u>: $F: S \rightarrow /p_1$

The horizon of the projected geometry is asymptotic to its respective meridians and appears separated from an \mathbb{R}^2 perspective. However, outward geometry is connected from an \mathbb{R}^3 perspective. Note the delta (discrepancy) that exists between higherdimensional \mathbb{R}^3 topology and projected (collapsed) lower coordinate points within their respective open sets.

Eq. 13:
$$(\mathbb{R}^{n-1}) \subset \mathbb{R}^n$$

Eq. 14: $(\mathbb{R}^{n-1}) < \mathbb{R}^n$

Distortions of meridians and orthodromes are given by:

 $\underline{\text{Eq. 15}}: x = \frac{\cos \phi \sin (\lambda - \lambda_0)}{\cos c}$ $\underline{\text{Eq. 16}}: y = \frac{\cos \phi_1 \sin \phi - \sin \phi_1 \cos \phi \cos (\lambda - \lambda_0)}{\cos c}$ $\underline{\text{Eq. 17}}: \text{ where } \cos c = \sin \phi_1 \sin \phi + \cos \phi_1 \cos \phi \cos (\lambda - \lambda_0)$

A similar gnomonic projection can be extended from an \mathbb{R}^4 hypersphere onto two spheres, as well as from an \mathbb{R}^5 hypersphere onto two \mathbb{R}^4 hyperspheres. These geometries are separated in their mutual \mathbb{R}^n space. However, their geometry remains connected in their higher \mathbb{R}^{n+1} space, with similar discrepancies and distortions.

6. PARAMETRIC VALIDATION REINFORCEMENT LOOPS Parametric Validated Reinforcing Loops (PVRL)

This section proposes the mechanism which delineates between each \mathbb{R}^n phase state to be PVRL: an iterated process of conscious binary gnomonic mapping of higherdimensional topology onto biased eigenstates (and subsequent propagation within the quantum field). At each iteration, symmetry becomes more broken, and geometric parameters become more constrained. "Consciousness as a fundamental unit of interaction with the Higgs field" is proposed as a derivation from the ubiquitous effects of consciousness in human experience, measurements, and experiments. The same dynamics (polarity, separation, proto-self-reinforcement, power-law distributions, and propagation) resulting from conscious interaction (PVRL) are congruent with dynamics observed in both QFT and GR.

PVRL feedback loops are reinforcing, as opposed to control loops which maintain a set point.⁴ The interactions tend to escalate

and deviate as a consequence of the discrepancies between their respective topologies. (Note that this same basic format is ubiquitous throughout nature.) PVRL interacts with the quantum field, through propagation (from QFT to GR) to increasing scales, with subsequent emerging vector spaces (including familiar classic space).⁵ The basic components are shown in Figure 8.

A and B can be regarded as rational agents in classic spacetime, however on a more fundamental level as operators in a vector field. Their inherent discrepancy cannot be resolved within the emerging \mathbb{R}^n space. Thus A's perception/interpretation of B is always skewed, and vice-versa. Consequently, reciprocating responses (following the



FIG. 8: Parametric Validation Reinforcement Loops.

principle of minimum energy to maintain a state of homeostasis) increase exponentially (equation 21) in a vicious cycle. A functional space emerges from homogeneous topology which can be represented as divergence in a vector field.

Note each partial derivative corresponds to n of \mathbb{R}^n space:

Eq. 18:
$$div \, \vec{v} = \nabla \cdot \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} + \dots \frac{\partial v_n}{\partial x_n}$$

A density emerges between agents with similar topologies, represented as negative divergence. The amount of energy required to ascend the resulting vector space is represented as the gradient.

$$\nabla f(x_1, x_2, \dots x_n) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \dots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

Decreasing Parameters of Recursive PVRL

Equation 21 describes the exponential development of PVRL. Dynamic field interactions (divergence, convergence and bifurcation) are subsequently propagated through alignments of charges. The inevitable result of such recursive exchanges is a decline of information (topology) with each iteration, forming a power-law distribution



FIG. 9: Typical Pareto distribution from Gaussian, after multiple exchanges.

with associated increased entropy.⁶ As scales increase, spacetime, symmetry, and dimensionality are greatly diminished (See figure 9).

Power law of recursive exchanges $p(x) = Cx^{\alpha} = \frac{C}{x_{\alpha}}, \text{ for } x \ge x_{min}$ Normalization for $(\alpha > 1)$ $1 = \int_{x_{min}}^{\infty} p(x)dx = C \int_{x_{min}}^{\infty} \frac{dx}{x^{\alpha}} = \frac{C}{\alpha - 1}x_{min}^{-\alpha + 1}$ $C = (\alpha - 1)x_{min}^{-\alpha + 1}$

Power law probability function (PDF)

$$p(x) = \frac{\alpha - 1}{x_{\min}} \left(\frac{x}{x_{\min}}\right)^{\alpha}$$

Per the Entropy-Power Inequality:

Eq. 19: $exp(2h(X+Y)) \ge exp(2h(X)) + exp(2h(Y))$

X and Y are independent real-valued random variables, and h(X) is the differential entropy of the PDF.⁷ This exchange and distribution can be conceptualized as similar to a tennis match as player A dominates player B. With increasing gap between them, the subordinate player loses control, position, and accuracy, to a point of failure. The resulting field dynamics are the "reality" of human existence. However, it is not an objective independent reality, but rather an emergent self-validated reality.

Self-Validated Realities

In order to understand PVRL and gnomonic-projected emerging pseudorealities, it is necessary to accept that within each \mathbb{R}^n space "objective observations" are profoundly biased and self-validated. The most familiar example of this phenomena is in the dynamics of dysfunctional partisan politics. The following real-world example is a hypothetical tribal rivalry. Note that PVRL in human interaction follows the same basic format as in figure 8, only more sophisticated. (Interpretations are biased projections, as in the Rorschach Test): Two tribes, A and B, are engaged in some dispute. Both tribes seek a state of homeostasis. However, their innate mutual discrepancy (represented as the complement of $(A \cap B)^c$), defies resolution (refer to figure 10):

Let g_i = their initial mutual discrepancy $(A \cap B)^c > 0$

Let α_i = their mutual initial assessment

Assume negative value $(a_i < 0)$

The following sequence emerges:

At iteration f:¹,

- $1.\,\mathrm{F}:\mathrm{B}\twoheadrightarrow/\mathrm{A}$
- 2. A views B's position with an initial bias of $-\overrightarrow{g_i}$
- 3. A responds with a shift of $+\overrightarrow{g_i}$
- 4. F : A →/B
- 5. B views A's position as the sum of the initial $-\overrightarrow{g_i} \overrightarrow{g_1} = -\overrightarrow{2g_i}$
- 6. B reciprocates with a shift of $+\overline{2g_i}$

Mutually, \propto is validated (self-validated).

 \propto increases to $\propto_i (1 + \frac{1}{k})^2$

At iteration f:²,

- 1. F : B →/A
- 2. A views B's position with a resulting bias of $-\overrightarrow{3g_i}$
- 3. A responds with a shift of $+\overrightarrow{3g_i}$
- 4. F : A →/B
- 5. B views A's position as the sum of $-\overrightarrow{2g_i} + -\overrightarrow{2g_1} = -\overrightarrow{4g_i}$
- 6. B reciprocates with a shift of $+\overrightarrow{4g_i}$

Mutually, \propto is validated (self-validated). \propto increases to $\propto_i (1 + \frac{1}{\nu})^3$

Through iteration *f*: ", [...] The discrete form of this process is expressed as:

Eq. 20:
$$div_n = \sum_{i=1}^n (4i-1)(\alpha(1+\frac{1}{k})^n)$$

However, PVRL regards these iterations to be occurring minutely at every moment, approaching a continuous function of:

Eq. 21:
$$div_n = (4n-1)\alpha_i e^{kn}$$

Generalized Dynamics of PVRL

This real-world narrative is just one emergent phase of PVRL. The same format of six-step sequence cycles are common throughout the cosmos, from QFT to GR, with increasing

sophistication and complexity. Notice two separate narratives driving this system expansion. A's self-validation of B's response is predicated upon B's self-validation of A's response. Neither narratives are independently derived. PVRL regards this self-validation to be = \forall of perceived reality, and the resulting dynamics (polarization, escalation, separation, bonding, gradient and negative gradient, force, diminished parameters of information, power-law distributions, and increasing entropy) as the functional field parameters at every emergence space in \mathbb{R}^n multispace. PVRL as the conscious fundamental units interacting within the Higgs field, provides an explanation for the



FIG. 10: Tribal dispute.

presence of an observer at wave collapse: The two sides of Einstein's field equations (matter and space) are connected in QFT \mathbb{R}^5 space, while observers in \mathbb{R}^4 spacetime collapse (partial map) information via PVRL. As the intermediate device of an optical plate is introduced into the experiment, validation is affirmed between the two scales, and the loop is completed.

7. SUMMARY AND CONCLUSION: CONNECTIONS IN \mathbb{R}^n SPACE

This theory models GR as a gnomic projection (of higher dimensions) onto a flattened disk, with associated meridian distortions, including the cosmic event horizon as the projected asymptote. A fascinating corollary is that connections in cosmology are reflected in most every classroom wall. The common world atlas (with a Mercator projection at 82S and 82N), which places the East Siberian Sea to be at opposite extremes from the Bering Sea, fails to account for their actual connection at the Bering Strait. This model proposes, by extension, that the outer extremes of GR are connected at the cosmic event horizon. This geometry is congruent with general relativity and Einstein's field equations. The only distinction is that the metric tensor indices (μ and ν) must transition (at the extreme boundaries of the entire scope) from 5 dimensions in QFT to 3 dimensions in GR. So, the amount of equations also must vary accordingly.

This model also suggests that all of matter remains connected in QFT, at \mathbb{R}^5 space. This can also provide an explanation for entanglement, as well as the fundamental forces: As nodes of connections are propagated (recursive topological mapping) through the Higgs field with increasing reduced parameters of d.o.f. in \mathbb{R}^n space (and increasing entropy), the intensity of force varies with distance per the inverse square law (described in section 6 as eigenvalues and vector field gradients). PVRL, as the conscious fundamental units interacting within the Higgs field, provides an explanation for the presence of an observer at wave collapse: The two sides of Einstein's field equations (matter and space) are connected in QFT \mathbb{R}^5 space, while observers in \mathbb{R}^4 spacetime collapse (partial map) information via PVRL. As the intermediate device of an optical plate is introduced into the experiment, validation is affirmed between the two scales, and the loop is completed. Note that such conscious feedback loops occur at extremely high frequencies during every brief period of conscious interaction, which defies human awareness. However, more measurable models can be extrapolated from conscious interaction in economics, social media, and politics. Perhaps the most obvious extrapolation is in the extreme case of dysfunctional partisan politics, where the dynamics of "self-validated realities" are clearly demonstrated: polarization, escalation, separation, bonding, gradient (and negative gradient) force, diminished parameters of information, power-law distributions and increasing entropy. Analyzing these "realities" reveals a truism that reality is predicated upon previous assumptions and is recursively self-validated (infinitum).

8. GLOSSARY OF TECHNICAL TERMS

- Backward causation: Any of several hypothetical phenomena or processes that reverse causality, allowing an effect to occur before its cause.
- Casimir plates: Uncharged conductive plates in a vacuum.
- Cosmic event horizon: The largest comoving distance from which light emitted now can ever reach the observer in the future.

- DeBroglies λ: The de Broglie wavelength of light and electrons.
- d.o.f.: Degrees of freedom.
- Entropy: A thermodynamic quantity representing the unavailability of a system's thermal energy for conversion into mechanical work, often interpreted as the degree of disorder or randomness in the system.
- Galaxy rotation curve: A plot of the orbital speeds of visible stars or gas in that galaxy versus their radial distance from that galaxy's center.
- Gnomonic Mapping: A planar perspective projection viewed from the center of the globe.
- GR: General relativity.
- Higgs field: A field of energy that is thought to exist in every region of the universe.
- Hubble constant: The unit of measurement used to describe the expansion of the universe.

- PDF: Probability density function.
- QFT: Quantum field theory.
- \mathbb{R}^n : Set of elements in an "n" dimensional space.
- Superposition: A property that states that for all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually.
- Vacuum energy density: An underlying background energy that exists in space throughout the entire universe.
- Wave function: A mathematical description of the quantum state of a system. The wave function is a complex-valued probability amplitude, and the probabilities for the possible results of measurements made on the system can be derived from it.
- Zero-point energy: The quantum state with the lowest possible energy.

NOTES

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