ACES: Articles, Columns, & Essays

A Periodic Table of Polyform Puzzles

by Kate Jones, RFSPE

What are polyform puzzles? A branch of mathematics known as Combinatorics deals with systems that encompass permutations and combinations. A subcategory covers tilings, or tessellations, that are formed of mosaic-like pieces of all different shapes made of the same distinct building block. A simple example is starting with a single square as the building block. Two squares joined are a domino. Three squares joined are a tromino. Compare that to one proton/electron pair forming hydrogen, and two of them becoming helium, etc.

The puzzles consist of combining all the members of a set into coherent shapes, in as many different variations as possible. Here is a table of geometric families of polyform sets that provide combinatorial puzzle challenges ("recreational mathematics") developed and produced by my company, Kadon Enterprises, Inc., over the last 40 years (1979-2019). See www.gamepuzzles.com.

- Polyominoes—squares
- Polycubes—cubes
- Polyiamonds—equilateral triangles
- Polyhexes—regular hexagons
- Polytans—isosceles right triangles
- Polyrhombs—congruent rhombuses (rhombi)
- Polyrombiks—multi-size rhombi
- Polyocts—octagons paired with squares
- Polyrounds—circles and quarter arcs, convex and concave
- Polybends—combinations of quarter arcs
- Polyhops—squares on a hopscotch grid (rows offset by half a space)
- Polyores—golden triangles

The POLYOMINO Series

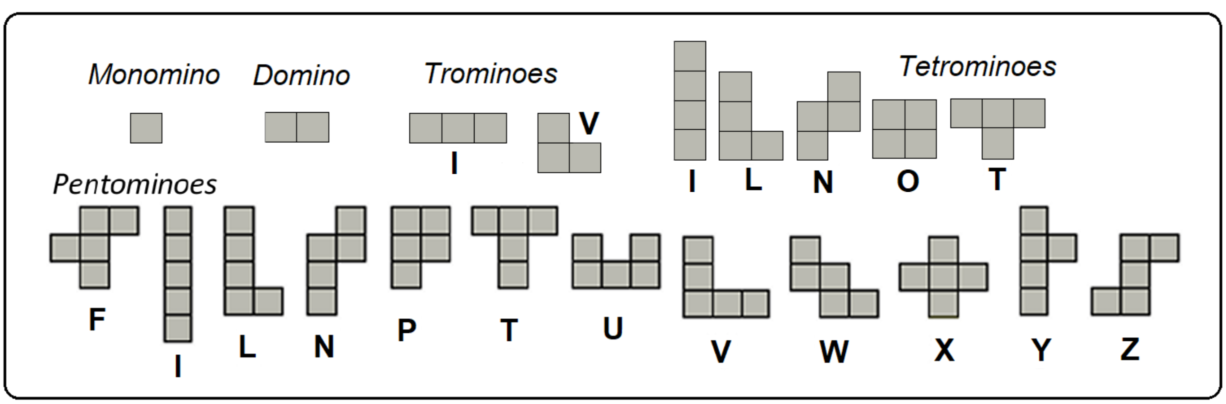

Figure 1: The polyominoes orders 1 through 5.

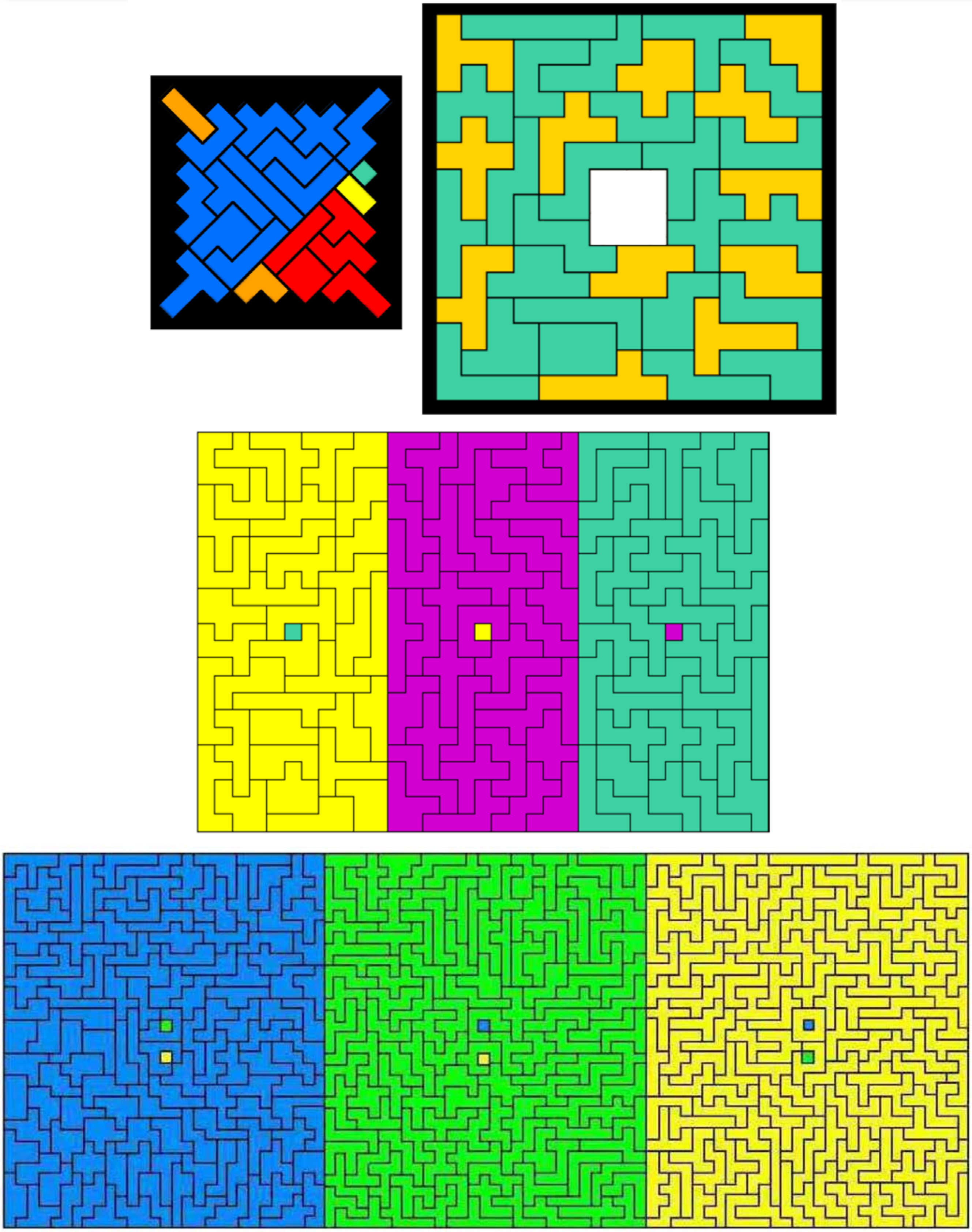

Figure 2: Polyominoes orders 1-5 (*Poly-5*); order 6 (hexominoes, *Sextillions*); order 7 (heptominoes); and order 8 (octominoes)—21, 35, 108 and 369 pieces, respectively. Polyominoes named by Solomon Golomb in 1953. Product names by Kate Jones.

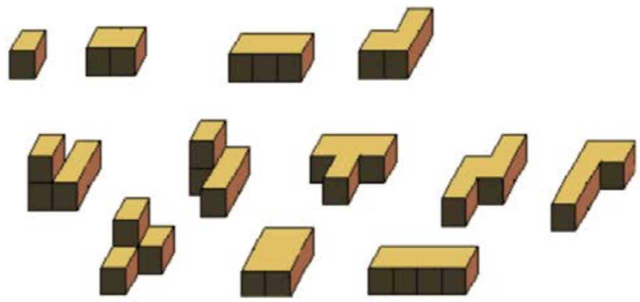

Figure 3: *Left*—Polycubes orders 1 through 4. *Right*—Polycubes order 5 (the planar pentacubes, Kadon's *Quintillions* set).

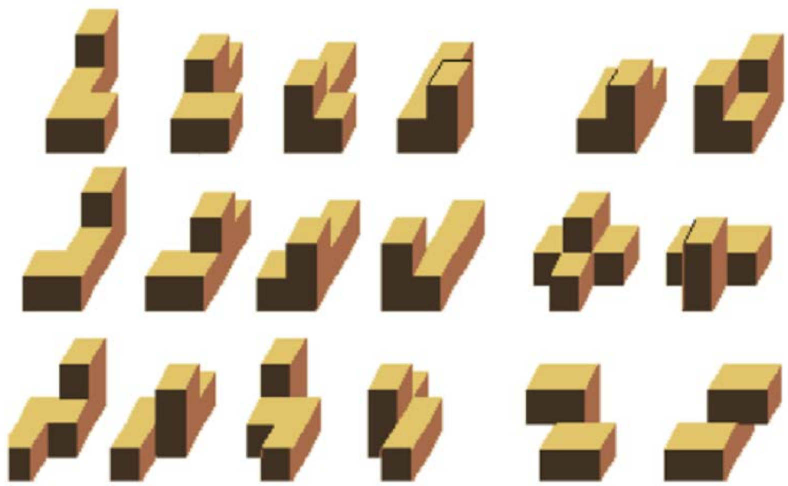

Figure 4: *Above*—the 18 non-planar pentacubes (*Super Quintillions*). *Below*—the 166 hexacubes.

The POLYIAMOND Series

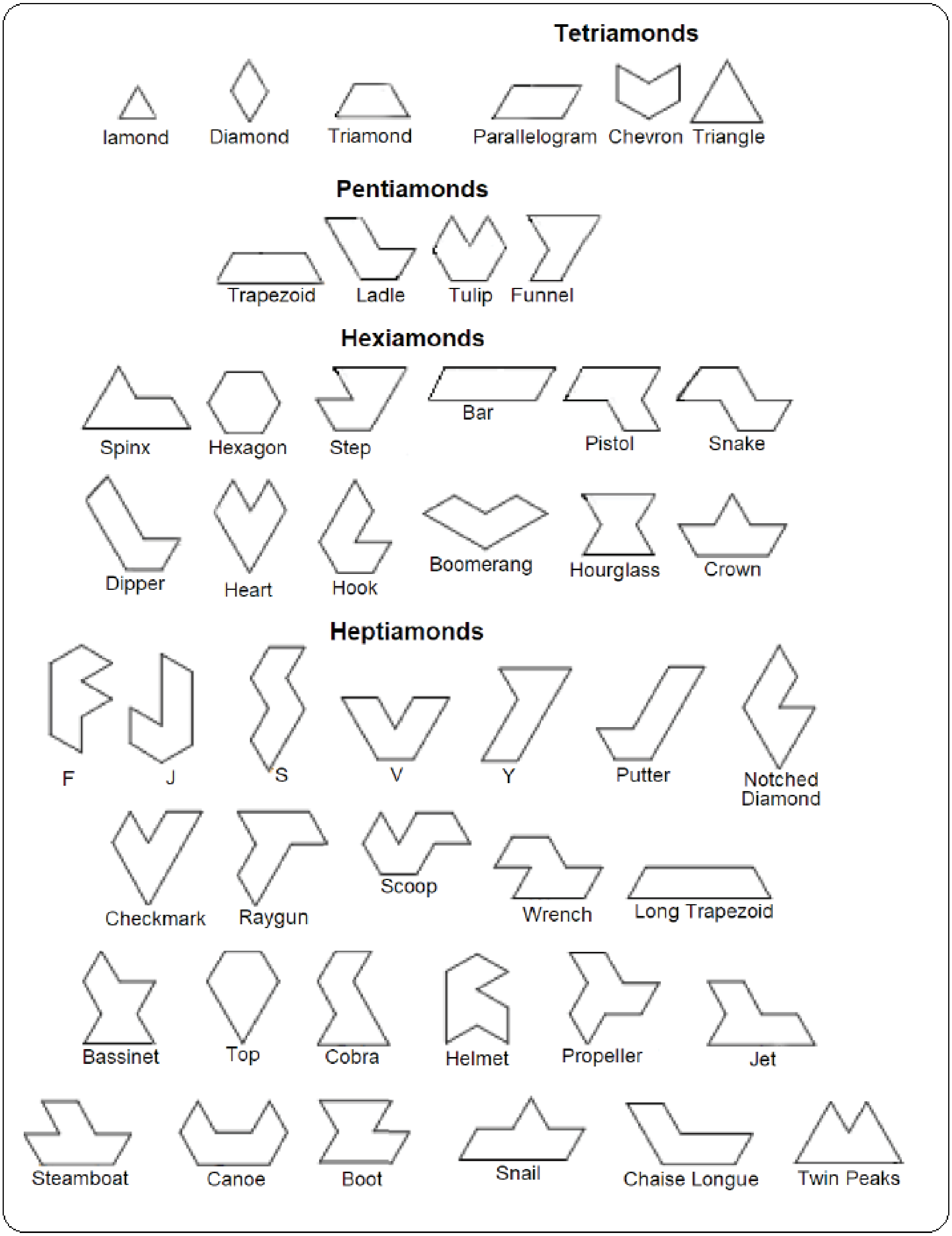

Figure 5: The polyiamonds orders 1 through 7.

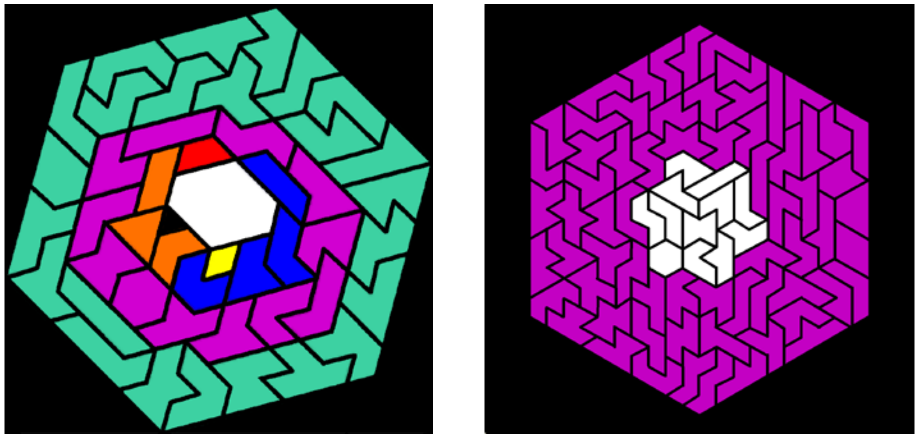

Figure 6: *Left*—polyiamonds 1-7 assembled. *Right*—the 66 octiamonds.

The POLYHEX Series

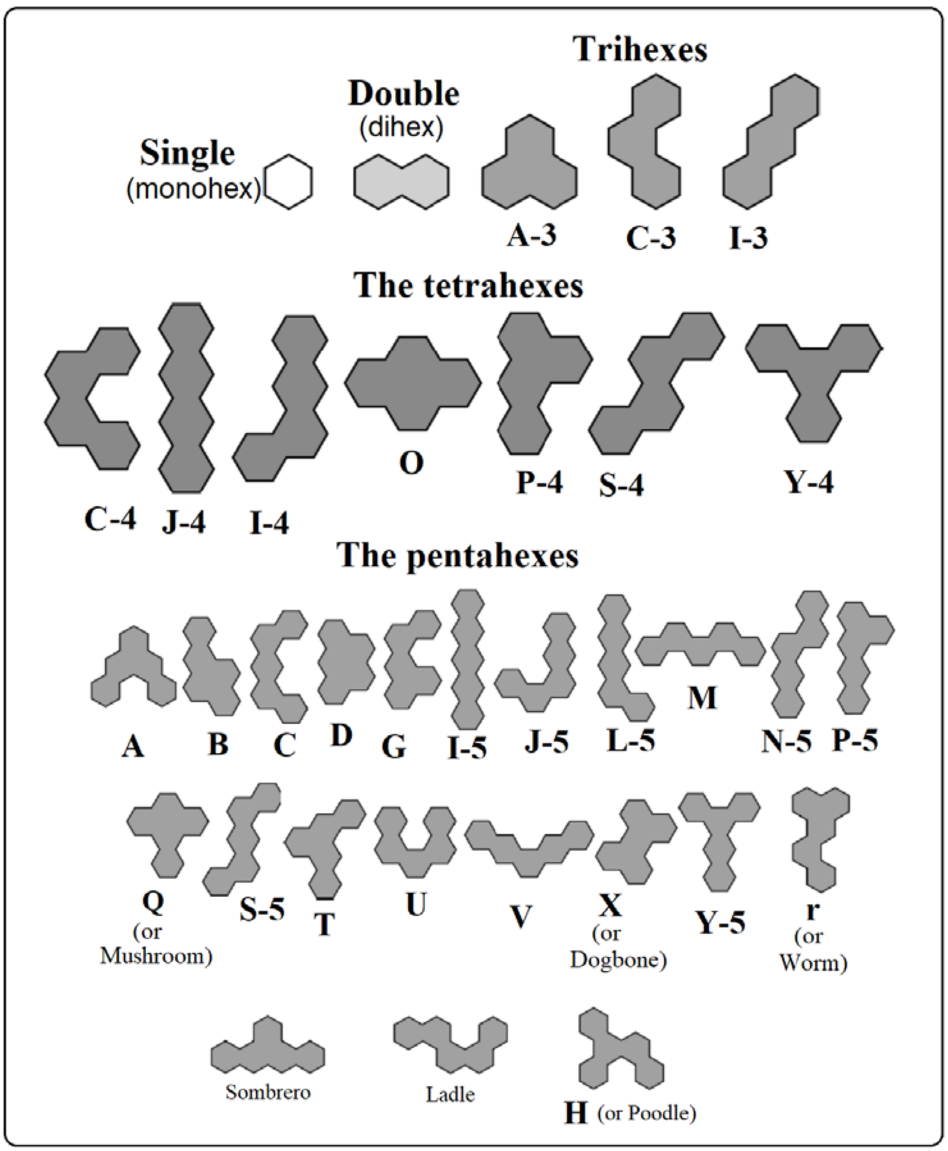

Figure 7: The polyhexes orders 1 through 5.

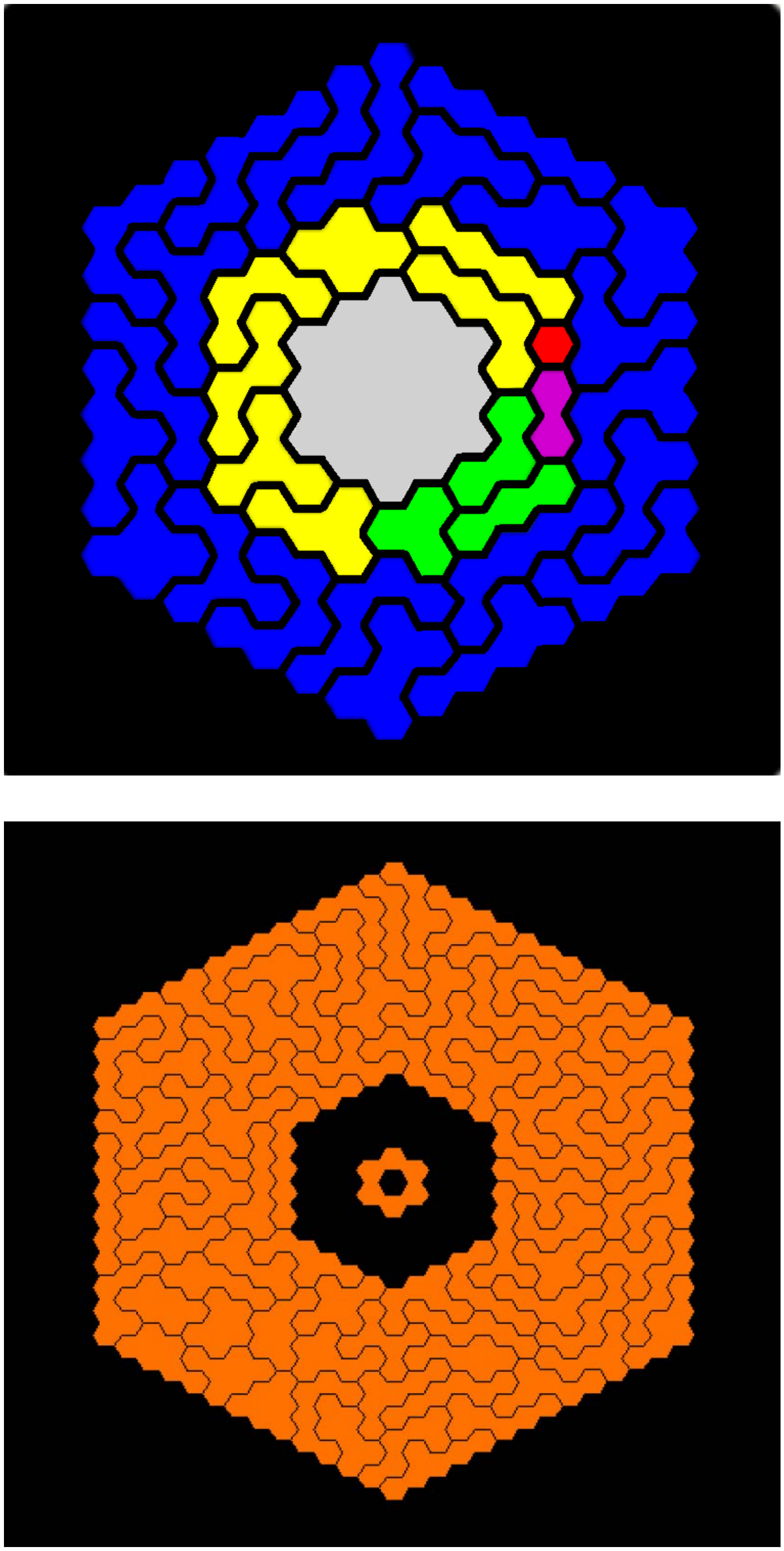

Figure 8: *Above*—the Polyhexes orders 1 through 5 assembled. *Below*—the 82 hexahexes.

The POLYTANS Series

Polytans are named after the centuries-old Chinese set known all over the world as *tangrams*. The original tangrams set has seven pieces: two isosceles right triangles of unit size, a parallelogram consisting of two unit triangles, a square, an order-2 triangle, and two triangles of size 4. These are not a pure combinatorial group, since there are duplicates of the order-1 and order-4 triangles, and there are no order-3 tiles. Nevertheless, their popularity is unequalled, and countless geometric and fanciful patterns result from their assembly.

The *polytans* sets go from unit to any size a puzzler dares to embark upon. We stop at size 6 as the limits of manageability in our *Tan Tricks* ensembles.

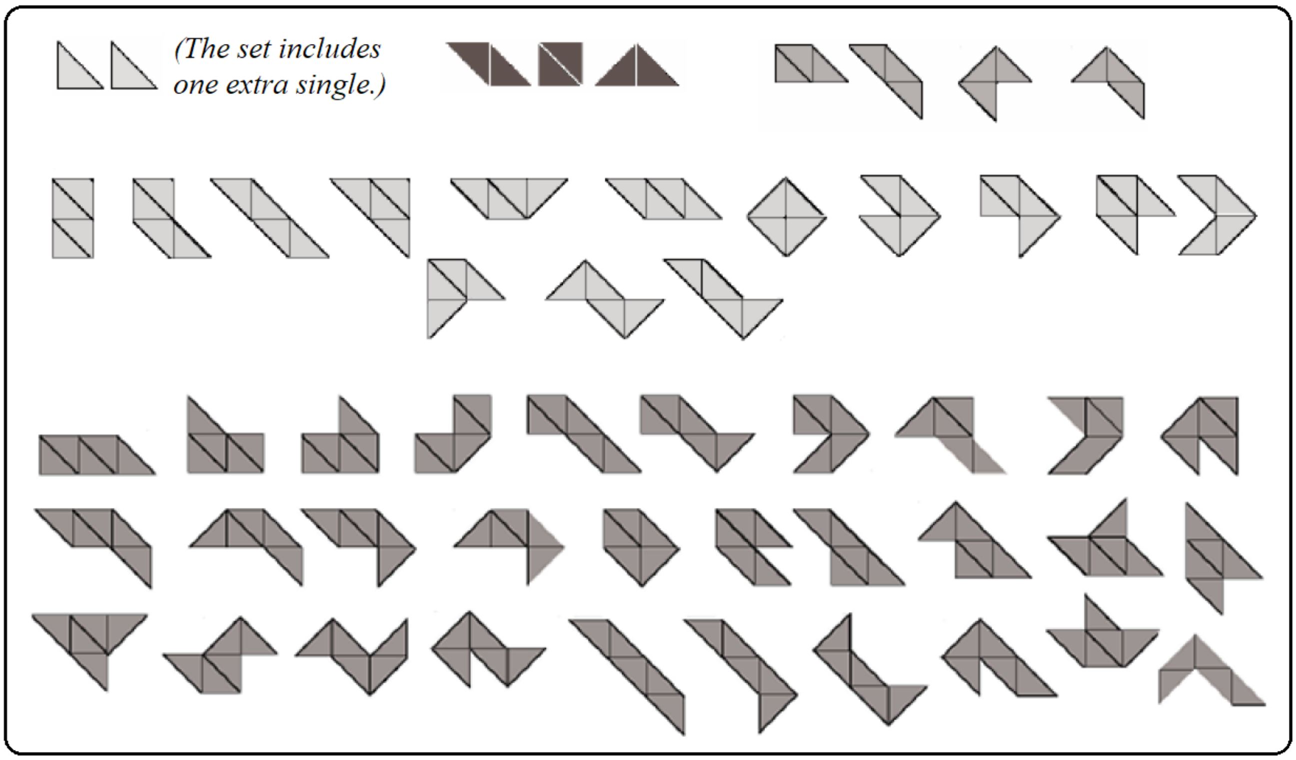

Figure 9: The polytans of orders 1 through 5 (1, 3, 4, 14, and 30 shapes, respectively).

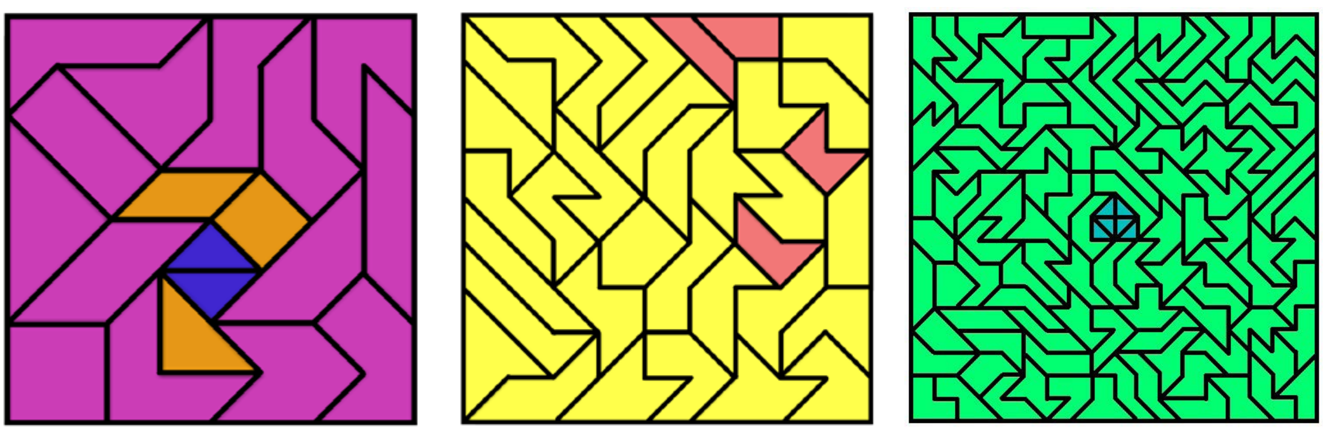

Figure 10: *Left to right—Tan Tricks I*, *Tan Tricks II* (orders 1-5), and the 107 hexatans of *Tan Tricks III.*

The POLYRHOMBS Series

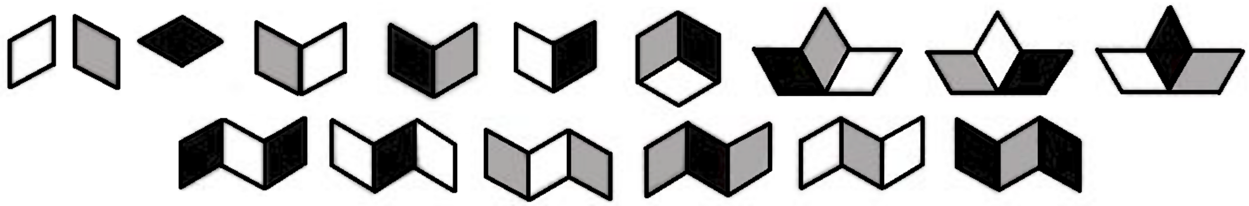

Figure 11: Polyrhombs orders 1 to 3 combined with 3 colors, making each tile unique and providing a complete set of all possible combinations that form a cubic grid and solve hundreds of polyhex figures.

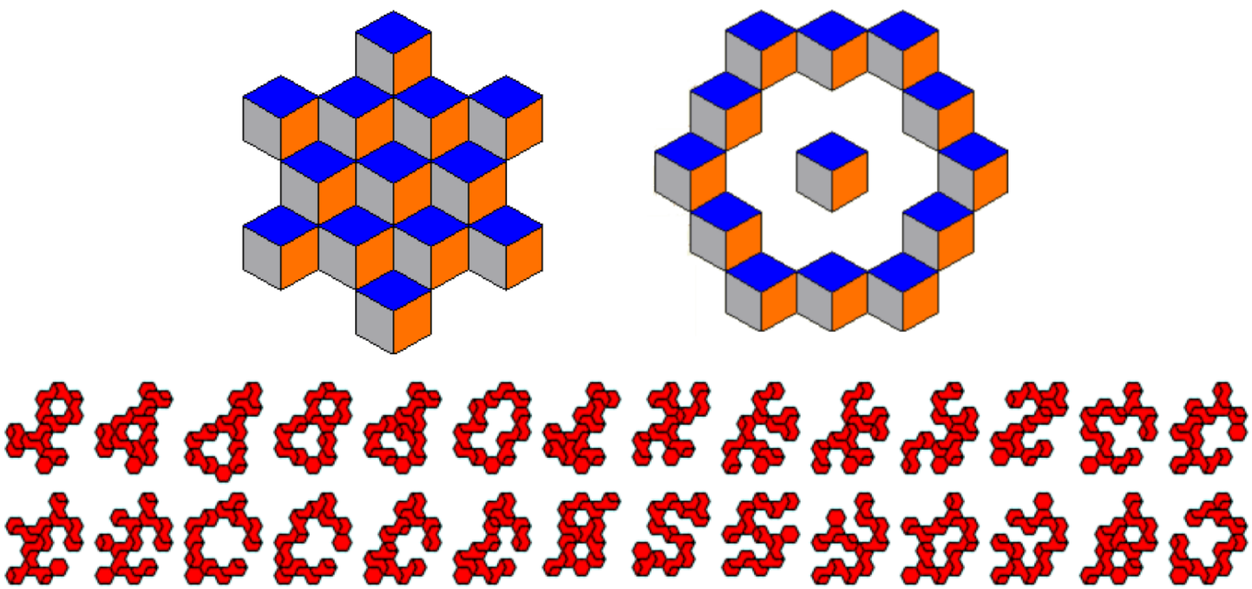

Figure 12: A small selection of cube-patterned figures, part of the *Cubits* puzzle set.

The POLYROMBIKS Series

Rombiks are the invention of the mathematician Alan Schoen, an associate of R. Buckminster Fuller in Carbondale, IL. Alan discovered that any even-sided polygon (six or more sides) could be dissected into rhombi—shapes of equal lengths of parallel sides with differing sizes of angles. Each polygon would produce its own characteristic number of different rhombi. A 16-sided polygon divides into 4 distinct sizes of rhombi. These could then be paired in all the possible different concave "twins" (Figure 13), and the resulting set of pieces would exactly tile the original polygon in a great many different ways, making for a fascinating puzzle, titled *Rombix*. A simpler set we developed on the same principle is *Rombix Jr.*, an octagon dissection. We also went one step further, to 24 sides and 6 colors: *Rainbow Rombix* shown in Figure 14.

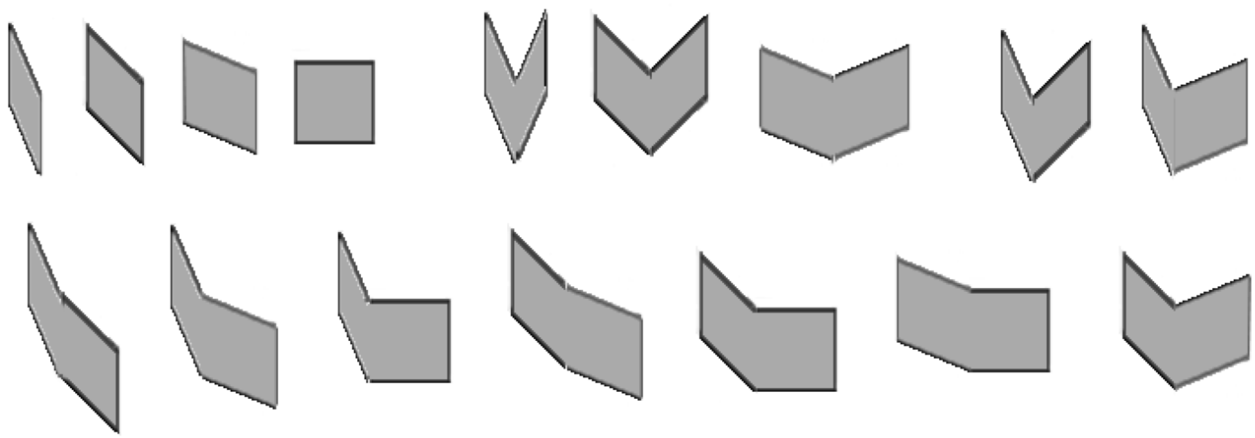

Figure 13: The 16 tiles that form the 16-sided *Rombix* set of all single and concave twin rhombi.

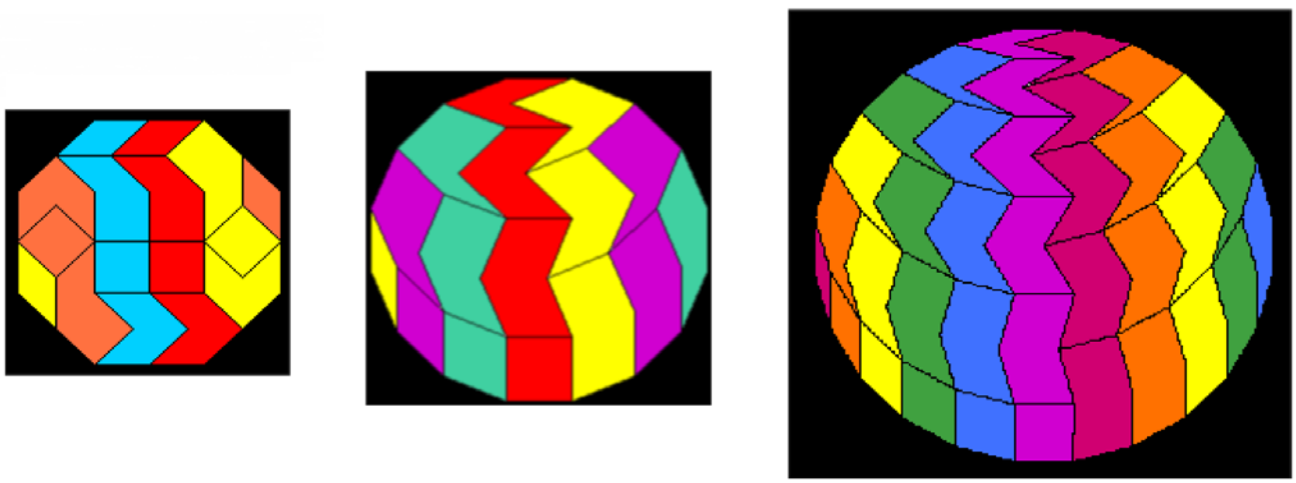

Figure 14: *Left to right—Rombix Jr.*, *Rombix*, and *Rainbow Rombix*.

The number of unique single rhombs determines the number of colors, and each color has the same "inventory" of component rhombs—a unique mathematical phenomenon. Each set has thousands of solutions.

The POLYOCTS Series

Octagons, unlike squares, triangles, and hexagons, cannot tile the plane without leaving holes. Pairing them with squares as fillers allows many beautiful mosaic patterns. The *Ochominoes* set consists of diocts—pairs of octagons joined like a domino. Attaching squares to their various sides, from 1 to 6 (Figure 15), generates 24 unique tiles that lend themselves to a dizzying variety of puzzle assemblies.

Single octagons can also be turned into elemental variations by attaching colors to their sides in every combination, with square islands snuggled between them. We made them in several styles, where the colors on every tile need to match their neighbors. See them in Figure 16.

Joining more than two octagons into a single piece is a story for another day. The complexity grows.

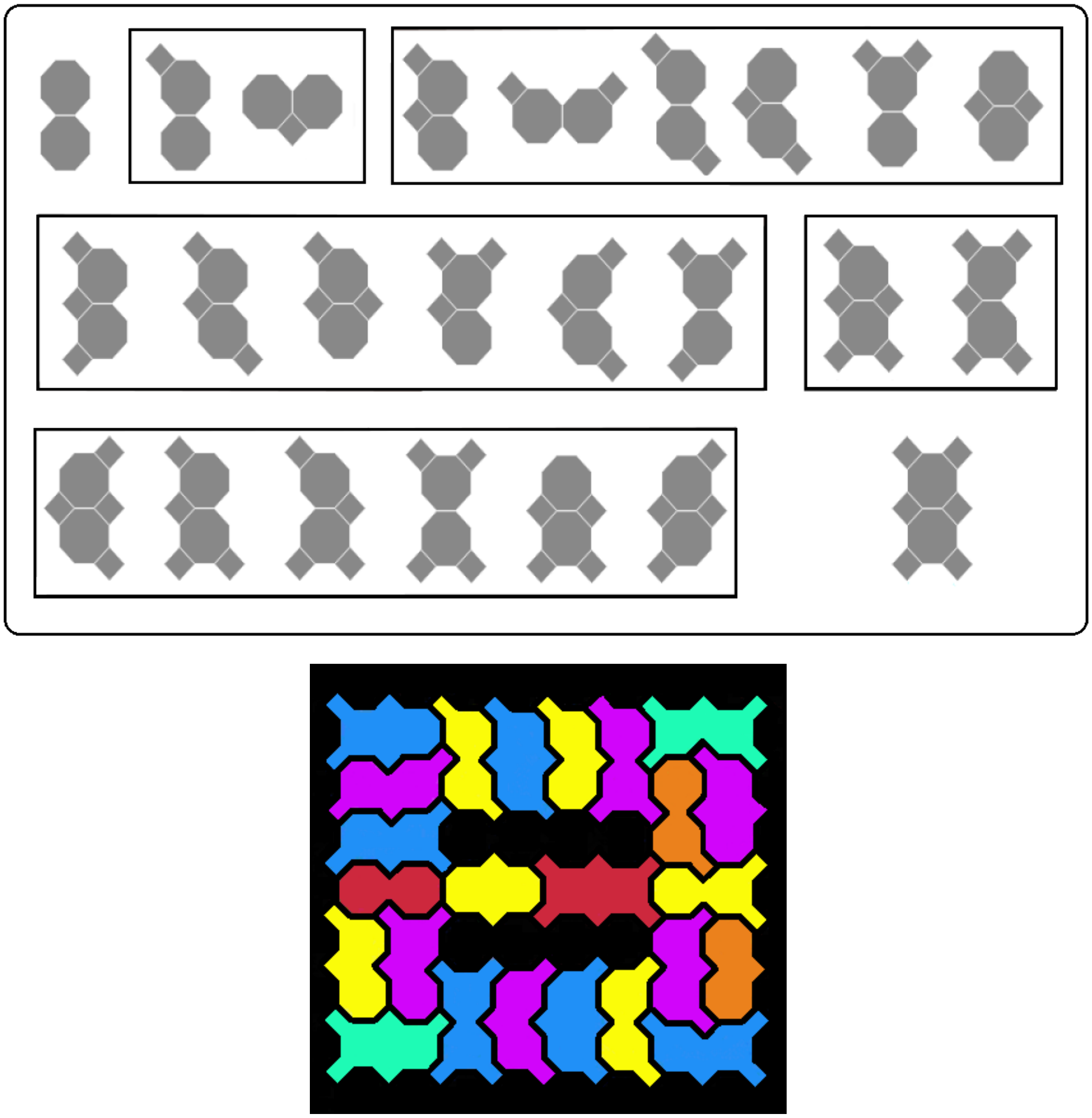

Figure 15: The 24 tiles of the *Ochominoes* set, grouped by how many squares are attached, and one of the many ways to form them into a rectangle. Each group has its own color.

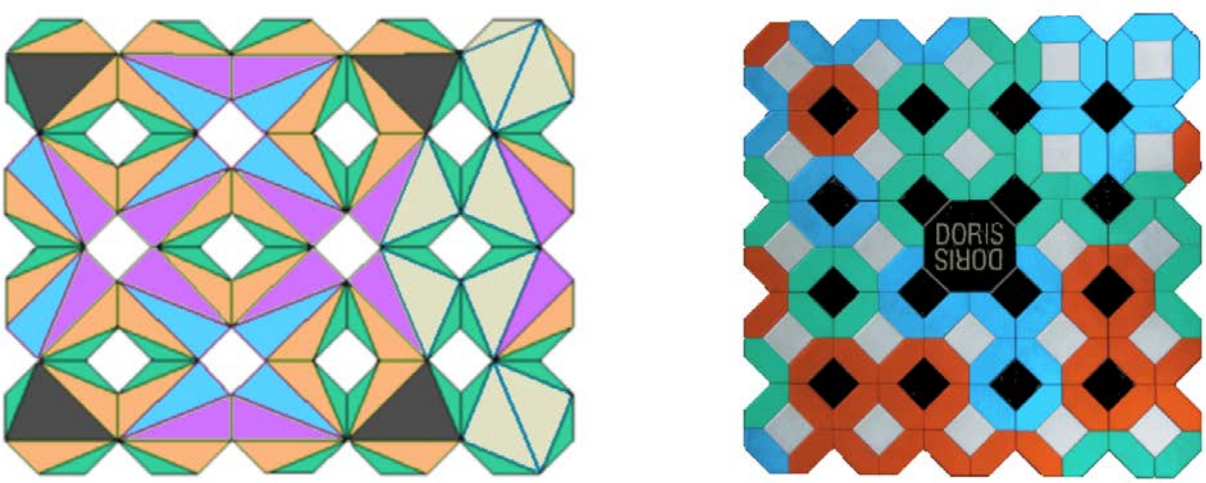

Figure 16: *Left—Triangule-8* with each octagonal tile inlaid differently with 6 triangles in 6 colors. *Right—Doris* with edge-colors in every combination of 3 colors.

The POLYROUNDS Series

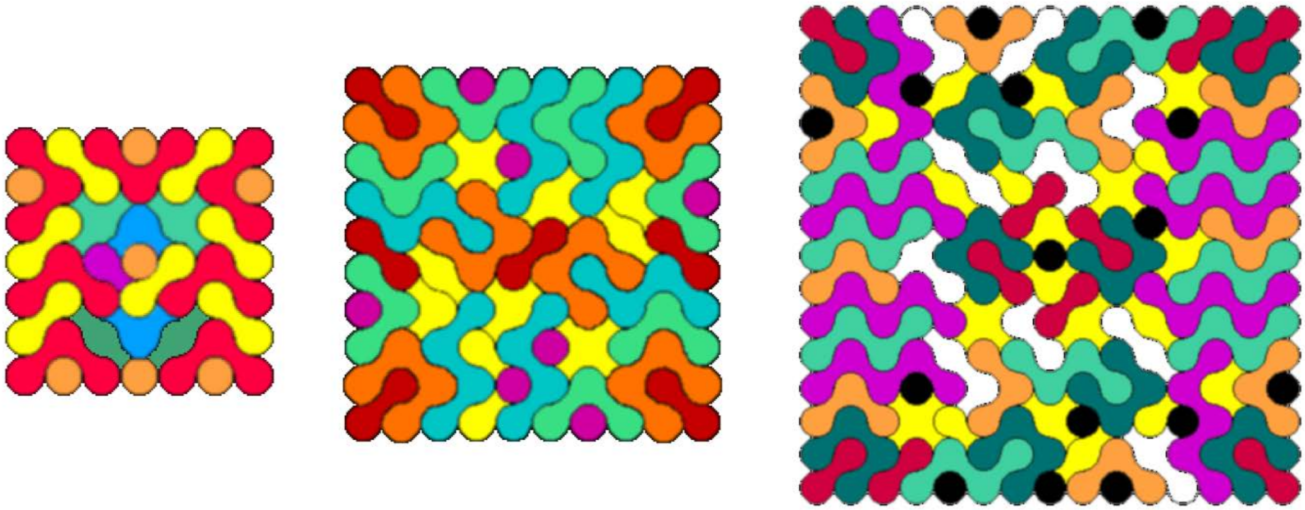

Figure 17: *Left to right—Roundominoes*, *Super Roundominoes*, *Grand Roundominoes*; orders 1-3, 1-4, and 1-5, respectively.

Tiles are formed of circles with concave circles ("bridges") attached, where quarter arcs can be convex or concave. Up through order-6 has been explored, with a level of difficulty not recommending release to the public at large. This intertwining concept can extend to infinity.

The POLYBENDS Series

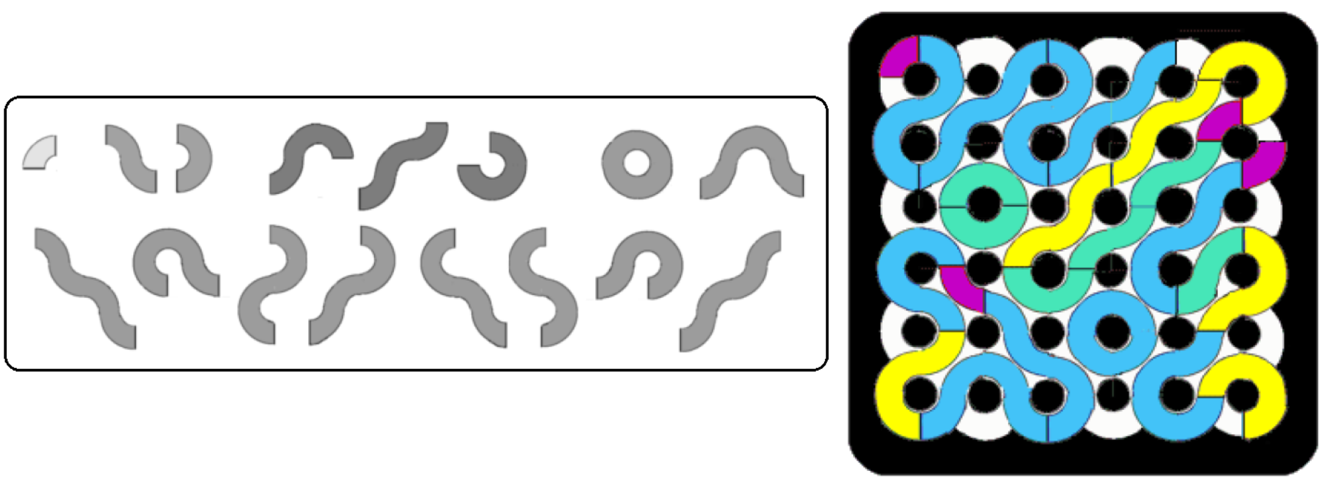

Figure 18: Quarter arcs of orders 1 through 4 yield 12 shapes that can form paths and loops.

Our *ChooChooLoops* set contains several duplicates to fill a 6x6 enclosure with 36 islands around which the path sections can wind. The concept can be extended endlessly, to the limits of human endurance. We may yet go to order-5 someday.

The POLYHOPS Series

Named after the hopscotch grid of everyone's childhood, the shapes of polyhops are formed of squares that shift by half a space on every row, like brickwork. Our *Hopscotch* set has orders 1 through 4 (Figure 19). That the order-4s produced the large number of 16 different shapes was a surprise and dissuaded us from pursuing order-5, at least for the time being

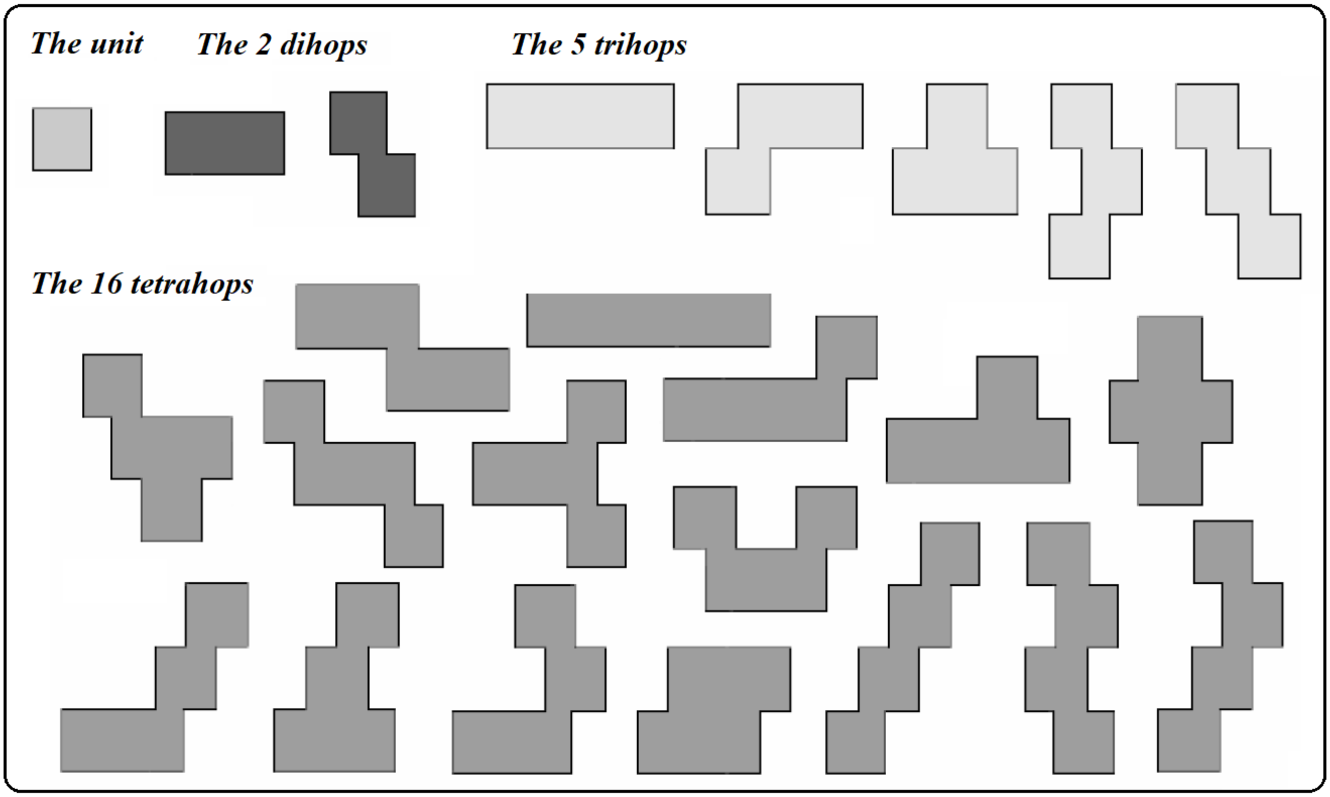

Figure 19: The polyhops of order 1 through 4 that can be plotted on a hopscotch grid.

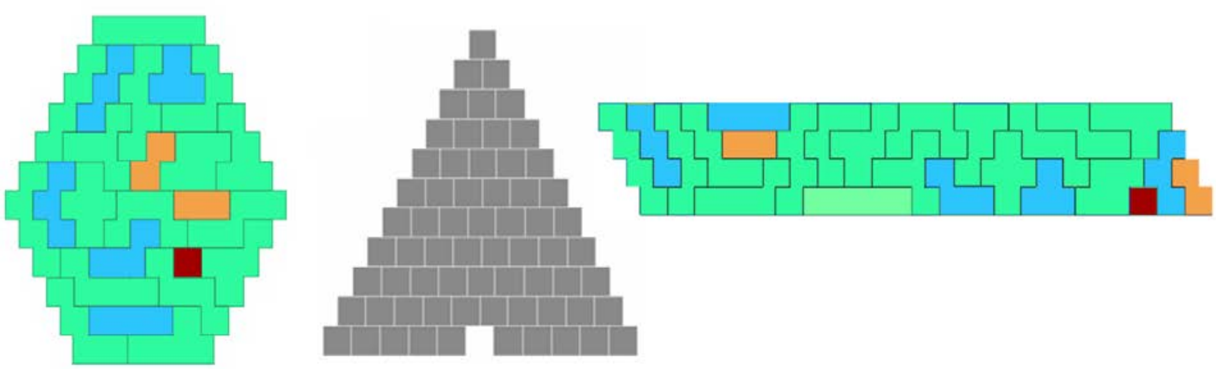

Figure 20: A few of the hundreds of *Hopscotch* constructions.

The POLYORES Series

Polyores are the invention of Jacques Ferroul of France. He named them *–ores* as meaning "gold"—the tiles are composed of all the possible combinations of the two golden triangles, in orders 1, 2, and 3. Each subgroup has its own color.

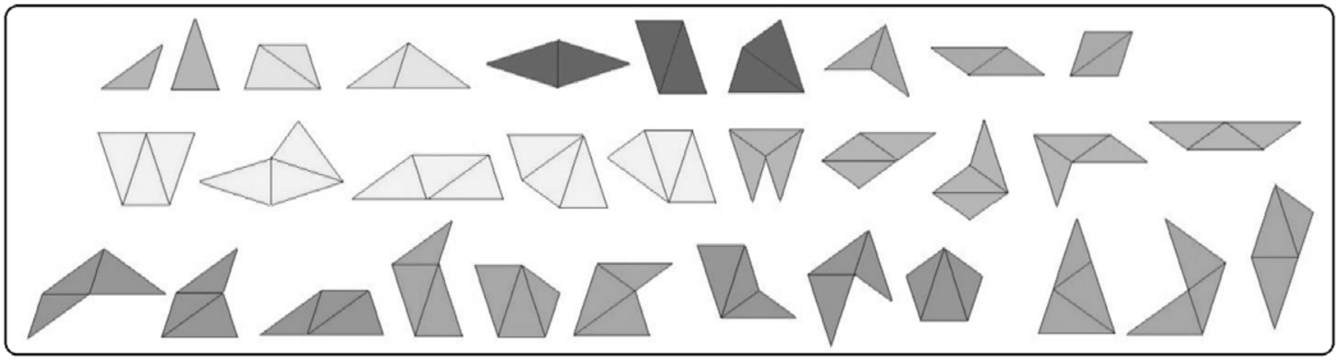

Figure 21: The 32 shapes formed of the two golden triangles of angles 72-72-36 degrees and 36-36-108 degrees in every combination of 1, 2, and 3 parts.

The lengths of the sides of the triangles are in proportion of the golden ratio, *phi*, the infinite decimal 1.618… Hence the name, golden triangles. They can form pentagons, decagons, stars (Figure 22), and many shapes still to be discovered. Figure 23 shows how they form our embodiment of this set as *La Ora Stelo.*

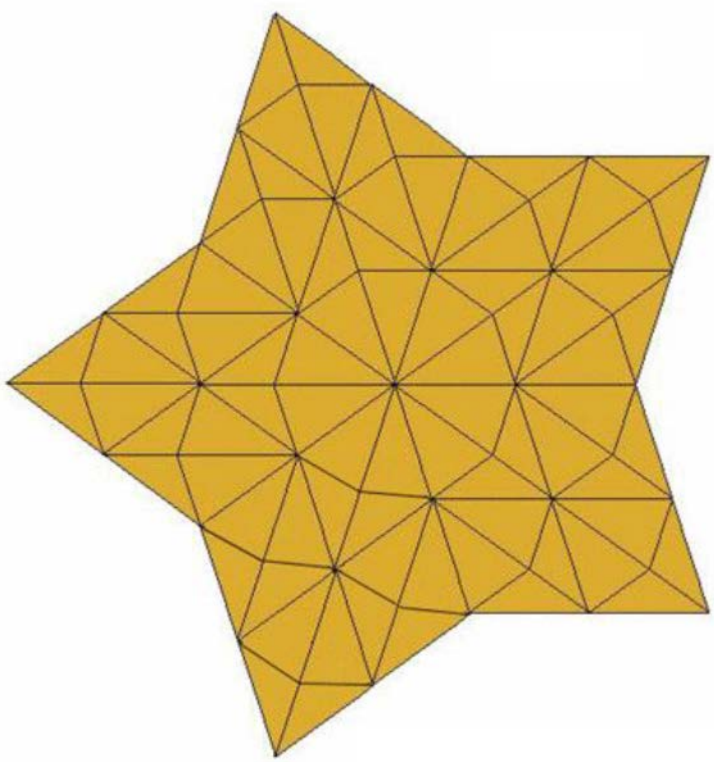

Figure 22: A grid of golden triangles forming a star.

Figure 23: *La Ora Stelo*—orders 1 through 3 polyores forming a pentagon, with 5 extra golden triangles (black area) as fillers.

Conclusion

This enumeration of polyform puzzles based on their topological essence is not closed. As Tom Lehrer said of the elements and might have said of this project, these are the ones of which news has come to Harvard, while many more are still waiting to be discovered. The essence is that we begin with the singularity and theoretically can expand to infinity. Each chain of expansion is a universe onto itself, with a periodic sequence that builds with an ascending continuity. And at every stage, one can dissect back to its origins. Each stage grows from the preceding level, demonstrating the fundamental principle of evolution.

There is something in the human mind that finds order and progression beautiful, innately pleasing, and encouraging to take the next step. It is how we build the future, on well-understood premises and ever-growing wisdom. Combinations let us build without end. **Ω**