Notes on MOND

by Robert McKnight, DSPE

1. Introduction

The currently accepted theories of motion disagree with the observed velocities of certain stars and galaxies. When there is disagreement between theory and observation, the principles stated in the *scientific method* would have the scientist reject the theory. In this case, where observed velocities of stars and galaxies are greater than theory predicts, the majority of astronomers have adopted the opposite principle and have, instead, rejected the observations. These astronomers insist that nature is not what has been observed—not by a long shot. Their claim is that there is much more matter and energy in the universe than can be observed in the usual ways—perhaps almost 20 times as much. These astronomers have dubbed this matter and energy *dark* matter and *dark* energy, and they are currently spending much time and money in an effort to discover ways to observe them.

A minority of astronomers are doing what the scientific method demands and are considering revisions of the theory that would predict these high velocities in the circumstances that have been observed. One such revision of theory is called MOND, an abbreviation for modified Newtonian Dynamics.[1]

How should a member of the International Society for Philosophical Enquiry approach a subject of which he or she knows little? According to the name of our Society, the approach should involve enquiry of a special kind, that is, a *philosophical* enquiry. That can mean that the information sought should be of a fundamental nature. We need not seek the depth of knowledge of an expert, but the enquiry should culminate in some understanding of the basic principles. This essay is a result of such an enquiry into the subject of MOND.

One question we might ask is, “Why would anyone even consider modifying Newton’s dynamics?” After all, Newton’s laws have formed the basis for the engineering that has made our industrial society possible. They have even explained our solar system quite adequately. (There is some circularity in the explanation, but that will not be discussed here.) In this essay, the principal motive given for advocating a change in Newton’s laws is, again, that they fail to explain the velocities observed in the rotation of stars within galaxies and of galaxies within groups of galaxies.

These observations of velocities cannot be directly made with a long tape measure and a stopwatch. Instead, these velocities are only *inferred* by the Doppler effects on the spectra of the moving bodies, and there are astronomers who deny these velocities. However, these inferred velocities are generally accepted by most astronomers, and they are much greater than Newton’s laws predict. Instead of trying to correct the discrepancy by claiming the existence of vast amounts of invisible matter and energy, the advocates of MOND try to correct the discrepancy by modifying certain Newtonian *laws of motion* as described below.

As it has been explained to me by one source, there are two different and mutually exclusive modifications involved in MOND.[2] One is a modification of Newton's *Law of Gravitation*; the other is a revision of Newton's Second *Law of Motion*. In this essay, the two resulting theories will be referred to as MOND1 and MOND2.

2. Newton's Laws

Let's compare how things would be if our universe was either a Newtonian universe or a MOND universe. To orient ourselves, we will first discuss the Newtonian case. We will try to keep everything simple by supposing a two-body universe consisting of a $body_1$ of mass m_1 and a $body_2$ of mass m_2, which are each rotating in their respective circles of radius r_1 and r_2 about their own center of mass. They have constant velocities v_1 and v_2 and are at a constant distance D apart, so the equations for their radii are

(1) $r_1 = m_2D/(m_1 + m_2)$, and $r_2 = m_1D/(m_1 + m_2)$, both in meters.

In this two-body universe, there are only two accelerations acting on $body_1$; and, in the case of a Newtonian universe, these accelerations are

(2) $a_{G1} = Gm_2/D^2$, the gravitational acceleration, in m/sec^2, and

(3) $a_{C1} = v_1^2/r_1$, the centripetal acceleration, in m/sec^2.

But, by substituting equation 1 into equation 3, we have

(4) $a_{C1} = v_1^2/[m_2D/(m_1 + m_2)] = v_1^2(m_1 + m_2)/[m_2D]$ m/sec^2.

Since we will be comparing these results with the results for a MOND universe, let's compute how far apart the two bodies must be in order for the gravitational acceleration a_{G1} acting on $body_1$ to be very small; let's say equal to a_0, which we will take to be $1.2\text{x}10^{-10}$ m/sec^2. According to equation 2, this occurs if

(5) $D = [Gm_2/a_0]^{1/2} = [Gm_2/1.2x10^{-10}]^{1/2} = 0.913\text{x}10^5[Gm_2]^{1/2}$ meters.

For $body_1$ to be in the supposed rotation, we need $a_{G1} = a_{C1}$, or

(6) $Gm_2/D^2 = v_1^2(m_1 + m_2)/[m_2D]$, whence

(7) $v_1 = m_2\{G/[(m_1 + m_2)D]\}^{1/2}$, in m/sec.

Moving at velocity v_1, it takes $body_1$ ρ_1 seconds to complete a rotation where

(8) $\rho_1 = 2\pi r_1/v_1 = 2\pi[m_2D/(m_1 + m_2)]/[m_2\{G/[(m_1 + m_2)D]\}^{1/2}] = 2\pi D^{3/2}[(1/G)(m_1 + m_2)]^{1/2}$ seconds.

3. The Interpolating Functions

As will be explained in section 4, the modifications of Newton's laws are achieved by multiplying or dividing his laws by the interpolation function, $\mu(a)$. This function has two forms. The first is called the *simple interpolation function*. The second is called the *standard interpolation function*. (I do not know why they are called *interpolation* functions. I think the term "*modifying* functions" would be more appropriate.) The simple and standard interpolation functions are

$\mu_1(a) = a/(a + a_0)$, and $\mu_2(a) = [a^2/(a^2 + a_0^2)]^{1/2}$.

The quantity, a_0, is claimed to be a new constant of nature, a very small one. A value for it that has made the MOND functions fit the actual astronomical data the best is $1.2\text{x}10^{-10}$ m/sec^2.

Both of the interpolation functions are designed to approximate 1 when a is much larger than a_0. That way, there is little difference between a MOND and a Newtonian result when a is as large as it is on earth; that is, 9.80 m/sec^2. However, for values of a as small as a_0 or less, both functions approximate a/a_0, which can be much less than 1. This can greatly increase the effect of a law that is *divided* by an interpolation function.

I do not know how MOND advocates choose between $\mu_1(a)$ and $\mu_2(a)$. Below is a table of values of both functions for a few values of a.

a	$0.1a_0$	$0.2a_0$	$0.5a_0$	a_0	$2a_0$	$3a_0$	$10a_0$	$100a_0$
$\mu_1(a)$	0.091	0.200	0.333	0.500	0.667	0.750	0.909	0.990
$\mu_2(a)$	0.100	0.243	0.447	0.707	0.800	0.949	0.995	1.000

Table 1

4. The Modifications

Let's now consider the MOND case. We will equate the two accelerations as before, but now they will be modified accelerations where the modifications are the inclusion of the so-called *simple* interpolating function $\mu(a) = a/(a + a_0)$. These modifications, in m/sec^2, are either

(9) $a_{MG1} = Gm_2/[D^2\mu(a)]$ and $a_{MC1} = v_1^2/r_1$ (if MOND1) or

(10) $a_{MG1} = Gm_2/[D^2]$ and $a_{MC1} = \mu(a)v_1^2/r_1$ (if MOND2).

When the gravitational acceleration and the centripetal acceleration are equated, the same equation results in both the MOND1 and the MOND2 cases. That is, in both cases, we get

$Gm_2/[D^2\mu(a)] = v_1^2/r_1 = (m_1 + m_2)/(m_2D)$.

Clearly, we can use equations 5, 6, and 7, obtained in the Newtonian case, if we just replace G with $G/\mu(a)$ everywhere it appears. Thus, we have

(11) $D_M = 0.913\text{x}10^5\{[G/\mu(a)]m_2\}^{1/2}$ meters, and

(12) $v_{M1} = m_2\{[G/\mu(a)]/[(m_1 + m_2)D_M\}^{1/2}$ m/sec, and

(13) $\rho_{M1} = 2\boldsymbol{\pi}D_M^{3/2}\{[\mu(a)/G](m_1 + m_2)\}^{1/2}$ seconds.

5. Comparing the Two Universes

For our comparison, we will assume that D_M is as given at equation 11 and that $a = a_0$, which makes the simple interpolation formula $\mu(a_0) = a_0/(a_0 + a_0) = 1/2$ and
the standard interpolation formula $\mu(a_0) = [a_0^2/(a_0^2 + a_0^2)]^{1/2} = (1/2)^{1/2}$.

Then, we get the following equations:

(14) Simple $D_M = 0.913\text{x}10^5\{[G/(1/2)]m_2\}^{1/2} = (2)^{1/2}D = 1.414D$,

(15) Standard $D_M = 0.913\text{x}10^5\{[G/(1/2)^{1/2}]m_2\}^{1/2} = (2)^{1/4}D = 1.189D$,

(16) Simple $v_{M1} = m_2\{[G/(1/2)]/[(m_1+m_2)D_M]\}^{1/2} = (2)^{1/2}v_1 = 1.414v_1$,

(17) Standard $v_{M1} = m_2\{[G/(1/2)^{1/2}]/[(m_1+m_2)D_M\}^{1/2} = (2)^{1/4}v_1 = 1.189v_1$,

(18) Simple $\rho_{M1} = 2\boldsymbol{\pi}D_M^{3/2}\{[(1/2\}/G](m_1+m_2^{1/2} = (1/2)^{1/2}\rho_1 = 0.707\rho_1$, and

(19) Standard $\rho_{M1} = 2\boldsymbol{\pi}D_M^{3/2}\{[(1/2\}^{1/2}/G](m_1+m_2)\}^{/2} = (1/2)^{1/4}\rho_1 = 0.841\rho_1$.

So, in a two-body MOND1 universe, if $a = a_0$, the bodies are about 41% farther apart, and $body_1$ moves about 41% faster than in a similar Newtonian universe. Also, $body_1$ takes only about 71% of the time to complete its orbit. These differences are more pronounced in the MOND2 case. As we see, these percentages are independent of how massive the bodies are.

It seems reasonable to believe that the orbital velocities, orbital periods, and—if the distance to the two bodies is known—the distance between the two bodies are observable. The velocities can be inferred from observation of Doppler shifts. The periods and distances of separation can be inferred from observation of positions. Therefore, let's now compute those three attributes for $body_1$ for different assumptions about the masses m_1 and m_2.

We will take G to be $6.66\text{x}10^{-11}\text{m}^2/\text{kgsec}^2$. The results are shown in three tables below, where there is a table for a Newtonian universe, a table for a MOND1 universe, and a table for a MOND2 universe. The assumptions about mass are that $m_1 = m_2$ and that m_2 has three different values. Column 1 in each table represents m_2 having the mass of a sun-sized star ($1.989\text{x}10^{30}$kg). Column 2 represents m_2 having the mass of an Earth-sized planet ($8.97\text{x}10^{24}$kg). Column 3 represents m_2 having the mass of a hydrogen atom ($1.67\text{x}10^{-27}$kg). Using equations 5, 6, and 7 (or 14, 16, and 18; or 15, 17, and 19), we obtain the tables below shown in both metric and English units.

SOME RESULTS FOR TWO-BODY UNIVERSES

Case 1: A Newtonian Universe

Attribute	Two Suns	Two Earths	Two Atoms
Distance Apart	$1.051X10^{12}$ km, or $6.531x10^{11}$ miles	$2.231x10^{9}$ km, or $1.386x10^{9}$ miles	$3.044x10^{-12}$ cm, or $1.199x10^{-12}$ inches
Orbital Velocity	$7.938x10^{3}$ km/hr, or $4.933x10^{3}$ mph	$3.659x10^{2}$ km/hr, or $2.274x10^{2}$ mph	$1.188x10^{-9}$ cm/sec, or $4.675x10^{-10}$ inch/sec
Orbital Period	$4.165x10^{8}$ hours, or $4.751x10^{4}$ years	$1.916x10^{7}$ hours, or $2.186x10^{3}$ years	$8.053x10^{-3}$ seconds, or $2.237x10^{-6}$ hours

Table 2

Case 2: A MOND Universe with Simple Interpolating Function

Attribute	Two Suns	Two Earths	Two Atoms
Distance Apart	$1.486x10^{12}$ km, or $9.235x10^{11}$ miles	$3.155x10^{9}$ km, or $1.960x10^{9}$ miles	$4.304x10^{-12}$ cm, or $1.695x10^{-12}$ inches
Orbital Velocity	$1.122x10^{4}$ km/hr, or $6.975x10^{3}$ mph	$5.174x10^{2}$ km/hr, or $3.215x10^{2}$ mph	$1.680x10^{-9}$ cm/sec, or $6.610x10^{-10}$ inch/sec
Orbital Period	$2.945x10^{8}$ hours, or $3.359x10^{4}$ years	$1.355x10^{7}$ hours, or $1.546x10^{3}$ years	$5.693x10^{-9}$ seconds, or $1.582x10^{-6}$ hours

Table 3

Case 3: A MOND Universe with Standard Interpolating Function

Attribute	Two Suns	Two Earths	Two Atoms
Distance Apart	$1.250x10^{12}$ km, or $7.765x10^{11}$ miles	$2.653x10^{9}$ km, or $1.648x10^{9}$ miles	$3.619x10^{-12}$ cm, or $1.426x10^{-12}$ inches
Orbital Velocity	$9.438x10^{3}$ km/hr, or $5.865x10^{3}$ mph	$4.351x10^{2}$ km/hr, or $2.704x10^{2}$ mph	$1.413x10^{-9}$ cm/sec, or $5.559x10^{-10}$ inch/sec
Orbital Period	$3.503x10^{8}$ hours, or $3.996x10^{4}$ years	$1.611x10^{7}$ hours, or $1.838x10^{3}$ years	$6.773x10^{-3}$ seconds, or $1.881x10^{-6}$ hours

Table 4

Do you think there is enough of a difference shown in the tables above to allow an observer to distinguish which case applies to our real universe? In our universe, there are many binary star systems that are widely separated from other bodies, so the "two-suns" situation may be approximated in real space. However, are there binary star systems in which the gravitational acceleration is as small as a_0; that is, where the distance between the two stars is hundreds of billions of miles? If so, could (would) they be observed? Their orbital periods may be as long as 47,510 years. Their observation would require a very patient astronomer.

Two isolated Earth-sized objects might be only one billion miles apart, and their orbital periods might be only 2,180 years, but would such objects even appear in a photograph? The idea of two isolated atoms seems even more problematic. When the two atoms are as close as 1.199×10^{-12} inches, would electromagnetic forces play a role?

To appreciate the magnitudes of the numbers in the tables above, it may be helpful to compare them with more familiar numbers. For instance, the distance of the nearest star to our solar system is about 24.9×10^{12} miles. The distance from the sun to Neptune is approximately 4.47×10^9 miles, and Neptune's orbital period is about 165 years. The radius of the orbit of a hydrogen atom's electron is about 0.5×10^{-8}. If this radius seems incredibly small, note that *Planck's length*—a length believed to be a minimum possible length—is approximately 10^{-35}.

It is fascinating to imagine that, while MOND advocates are searching for evidence to support their theory, it is discovered that, by turning on a certain shade of magenta light, dark matter becomes visible, and dark energy can be harnessed. Let's say it turns out that we here on Earth are wading around, up to our hips, in dark matter and energy. Suppose that we could learn how to fashion dark matter into window panes of a most transparent kind, and that we were able to convert dark energy into an inexhaustible supply of electrical power. Such discoveries would be judged as the biggest advances of physics in centuries.

6. Questions

According to equation 2, $a_{G1} = Gm_2/D^2$. But that is the *Newtonian* gravitational acceleration. Isn't the MOND1 gravitational acceleration equal to

$$a_{MG1} = (Gm_2/D^2)/[a/(a + a_0]?$$

Which acceleration do we plug into the interpolating function $\mu(a) = a/(a + a_0)$? Maybe it should be a_{MG1}. Acceleration is not something that can (easily) be measured. It's more likely to be calculated from its formulas. That is straightforward in the Newtonian case, but what do we do in the MOND1 case? Well, I think we might use the a, such that

$$a = [Gm_2/D^2]/[a/(a + a_0)], \text{ or}$$

$$[Gm_2/D^2][(a + a_0)/a] = a, \text{ or}$$

$$[Gm_2/D^2] = a\,[a/(a + a_0)] = [a^2/(a + a_0)], \text{ or}$$

$$[Gm_2/D^2](a + a_0) = a^2, \text{ or}$$

a^2 - $[Gm_2/D^2]a$ - $[Gm_2/D^2]a_0 = 0$.

Whence, by the quadratic formula,

$a = 1/2\{(Gm_2/D^2) \pm [(Gm_2/D^2)^2 + 4(Gm_2/D^2)\, a_0]^{1/2}\}$.

This would lead us to conclude that

$a_{\mathrm{M}} = 1/2\{a_N + [\, a_N^{\ 2} + 4a_N]^{1/2}\} > 1/2(a_N + a_N) = a_N$, where

a_M is the MOND1 acceleration, and a_N is the Newtonian acceleration.

In a MOND1 universe, for what D is the gravitational acceleration equal to a_0? Well, MOND1 gravitational acceleration $a_M = [Gm_2/D^2]/[a_M/(a_M + a_0)] = a_0$ when $[Gm_2/D^2]/[a_0/(a_0 + a_0)] = a_0$, or when $Gm_2/D^2]/[1/2] = a_0$, or when $a_0D^2 = 2Gm_2$, or $D_M = (2)^{1/2}[Gm_2/a_0]^{1/2} = 1.414D_N$. This is the relationship used at equation 14.

NOTES

1. Mordehai Milgrom, "A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis," *The Astrophysical Journal* 270 (July 15, 1983): 365-370, http://articles.adsabs.harvard.edu/full/1983ApJ...270..365M.

2. Written exchanges with Dr. Lawrence Kugler, retired mathematician who was a professor at the University of Michigan at Flint. He, in turn, consulted with his son-in-law, Dr. Stacy McGaugh, who is an astronomy professor at Case Western Reserve University and a principal analyst of MOND. **Ω**

"To explain all nature is too difficult a task for any one man or even for any one age.
'Tis much better to do a little with certainty and leave the rest for others that come after you."
—Isaac Newton